Is All Mathematics Applied Mathematics?

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What did C. F. Gauss mean when he proclaimed that "Mathematics is the Queen of the Sciences"? Unlike science, mathematics has no data. Mathematics, one of the fundamental human urges, determines precise relationships between abstract ideas using rigorous logical argument, and discusses these ideas in precise language. Mathematics is not the study of quantities; mathematics is the study of relationships and abstractions.

But is mathematics a queen like Victoria, or a queen like a bee? The queen bee is the focus of the hive, but her physiology and behavior resemble no other bee. And while it is correct to associate her with honey, she produces none. Nonetheless, without her there would be none.

This is the role of mathematics in science. We neither produce nor work with data, but the concept of science without mathematics is as absurd as the concept of honey without a queen bee.

Some see two kinds of queen. One queen is abstract and has no aim toward production; we call her "Pure Mathematics." The other is focused on the operation of the hive; we call her "Applied Mathematics." Most funding agencies separate the two. There is funding for *applied* mathematics, but a pure mathematical proposal sent to the DoE, DoD, or NIH will be returned unopened. NSA funds work in a few sub-disciplines. The only funding source for *pure* mathematics is NSF, whose focus is on support of large institutes and large groups of collaborators.

Still, while the pure mathematician's focus is not on producing honey, the combs are full.

Let's examine how this happens in some detail. The relationship between prime numbers and other whole numbers is intricate and delicate; some call it beautiful. Euclid established, by a clever proof, that there are an infinite number of prime numbers. Others later generalized the idea of primality to more abstract number systems, systems that are difficult or impossible to visualize. Deep questions about prime numbers remain unanswered, despite decades of effort.

When I was a graduate student these questions were regarded as hopelessly esoteric, even among mathematicians. Is this kind of mathematics too abstract for society to support? No: the results of humankind's wrestling with these abstract concepts are now built into your cell phone. Material that was recently too esoteric for the average PhD is now taught to senior undergraduates.

Support for pure mathematics is support for applied mathematics and support for science and technology. The external funding opportunities for pure mathematics are few, and in our abstract world we don't need dessicators or centrifuges or MRIs or accelerators, so the grants we get are small. We can't compete dollar-for-dollar with the other sciences. But without the queen there is no honey.